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clearly brought out, mechanical illustrations being freely used. The correlation coefficient should be developed as a product moment with the mean of the variables taken as an origin. Following this the students should be taught the theory of curve fitting by the method of moments and this theory should be illustrated by examples in the derivation of empirical formulas. The method of fitting curves by least squares need not be presented as it is not so widely applicable as the method of moments. The student should also have impressed upon him the importance of the idea of probable error and should be trained to work out the probable error of all such constants as mean, standard deviation, coefficient of correlation, etc.

The course just outlined would furnish a student in biometry with the proper mathematical foundation for his work; his mathematical needs are similar to those of the student of any natural science. One of the most common quotations among biometricians is that of Sir Francis Galton: "Until the phenomena of any branch of knowledge have been submitted to measurement and number it cannot assume the status and dignity of a science," and as we see more and more of the leading scientists taking this point of view it impresses upon us the growing importance of mathematics. The methods of teaching mathematics should develop accordingly; and if this present meeting succeeds in stimulating the teaching of mathematics to live up to its increased importance, it will indeed have been a success.

II. Complex Numbers in Advanced Algebra.

By H. E. Webb, Central High School, Newark, N. J.

In many electrical laboratories it is desired that the workers have a certain familiarity with vectors, and in particular with the exponential notation $e^{i\theta}$ Examination of texts in algebra most commonly used in schools fails to bring to light an elementary explanation of this notation. Certain texts in trigonometry give such an explanation, but usually in a rather intricate form, and without a clear statement as to what it is all about. The following brief outline is suggested as an addition to a course in advanced algebra for the fourth year in high school or the freshman year in college; in this it is desired to escape from exhaustive analysis of various points which seemingly of necessity must be brought into any *rationale* of this notation.

1. The rules of combination of complex numbers to be presented as an arithmetic of number pairs, *without reference to Argand's diagram.*
2. The sine and cosine of *real positive and negative numbers* to be defined by reference to a unit circle, without mention of other trigonometric functions, leading only to the facts that in a circle of radius r the corresponding lines are $r \sin \theta$ and $r \cos \theta$, and that $\sin^2 \theta + \cos^2 \theta = 1$ for all values of θ.
3. The development of formulas for $\sin (x \pm y)$ and $\cos (x \pm y)$.
4. The polar notation of a complex number, shown as follows: If $a + ib$ is

written $\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}+i\frac{b}{\sqrt{a^2+b^2}}\right)$, the real and imaginary coefficients may be regarded as the sine and cosine of some number, since the sum of their squares is unity. For the present the number is shown geometrically as an arc of a unit circle, the real coefficient being chosen arbitrarily as a cosine. Hence $a + ib$ may be written $r(\cos\theta + i\sin\theta)$.

5. DeMoivre's theorem, as usually presented, attention being called to the fact that in the case of $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, the argument seems to possess the characteristics of a logarithm, where n is any rational number.

6. The customary development of e and of e^x by the binomial theorem, as the limits of $(1 + 1/n)^n$ and $(1 + 1/n)^{nx}$ as $n \doteq \infty$. It is necessary at this point to make reference, of course, to convergency of series. It is not necessary to exhaust this topic, for the simple reason that the exponential series

$$e^x = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}}\cdots$$

looks to be convergent, since there is always a term beyond which the successive multipliers of the denominator exceed the corresponding multipliers of the numerator, for any value of x.

7. Since this series holds for all rational values of x, it will serve as a definition of the *meaning* in the case of an irrational exponent, and, as well, of an imaginary exponent, and also of a complex exponent, if $e^a \cdot e^{ib}$ is written e^{a+ib}. In the latter case $(a + ib)^n$ would require to be developed by the binomial theorem. It is *assumed* explicitly, though not proved, that the laws of rational exponents extend to complex exponents of powers of e.

8. Euler's theorem, established as follows:

Let $\cos 1 + i\sin 1 = e^k$, where k is a complex number to be determined.

Then $\cos 1 - i\sin 1 = e^{-k}$, since the product of these two equations, member for member, gives identity.

By De Moivre's theorem

$$\cos\theta + i\sin\theta = e^{k\theta},$$

$$\cos\theta - i\sin\theta = e^{-k\theta},$$

whence

$$\cos\theta = \frac{e^{k\theta} + e^{-k\theta}}{2}, \qquad i\sin\theta = \frac{e^{k\theta} - e^{-k\theta}}{2}$$

Now if $e^{k\theta}$ and $e^{-k\theta}$ are developed into series by the definition of e^x, and the series added and halved, term for term, it follows that

$$\cos\theta = 1 + \frac{k^2\theta^2}{\underline{|2}} + \frac{k^4\theta^4}{\underline{|4}}\cdots,$$

while

$$i\sin\theta = k\theta + \frac{k^3\theta^3}{\underline{|3}} + \frac{k^5\theta^5}{\underline{|5}}\cdots.$$

Dividing both members of the last equation by θ,

$$i\left(\frac{\sin\theta}{\theta}\right) = k + \frac{k^3\theta^2}{\underline{|3}} + \frac{k^5\theta^4}{\underline{|5}} \cdots.$$

As $\theta \doteq 0$, $(\sin\theta)/\theta \doteq 1$, and since k is constant,

$$k = i.$$

Substitution of i for k in the various formulas cited affords series development of $\cos\theta$ and $i\sin\theta$, in the first of which all terms are real, and in the second all imaginary; and also establishes the notation

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

9. If at this stage *and not sooner* the vector is defined the student will grasp perfectly the idea that such a directed line has two values, an algebraic value and an absolute value, which are different, in that its algebraic value depends upon its direction determined by the length of the arc of a unit circle described about the origin as a center.

It is the experience of the writer that in secondary school classes this development requires about as many days as he has topics outlined above, and that certain difficulties relating to the *value* of the ordinate are thereby avoided. A favorable opening is also provided for the teaching of hyperbolic functions, when this is desirable.

III. On Proofs by Mathematical Induction.

By E. T. Bell, University of Washington.

1. The best way to cure oneself of a crotchet is to confide it to some sympathetic listener. The crotchet in this note is one which has worried me since school days when I was induced to repeat the proof of the binomial theorem by mathematical induction. The same crotchet seems to trouble successive generations of freshmen, for occasionally one has obstinacy enough to balk at the magic formula "*and therefore the theorem is always true*," with which many authors conclude their proofs by recurrence. I hold that mathematical induction has no place in elementary teaching, particularly when such teaching strains at mathematical gnats, as in pseudo-rigorous presentations of the elementary theory of limits, the better to swallow logical camels such as some proofs of the binomial theorem or their equivalent quoted presently from Poincaré. This is the crotchet. In short, elementary teaching would be more convincing if it left rigor to that logistics which was Poincaré's bête noir.

2. It would be difficult to find a balder statement of the logical vice which characterizes many proofs by mathematical induction in the current text books, than the following extract from Poincaré's essay *On the Nature of Mathematical Reasoning*, in *Science and Hypothesis* (Halsted's translation, page 36, section IV).